Redefining Mathematics:
ULOGIC and the Irrelevance of Gödel in Dynamic Constructive Systems

Philosophical-Mathematical Impact of ULOGIC

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Resumen

This report develops a philosophical-mathematical discussion on the foundations of ULOGIC, a dynamic and constructive formal system that redefines the role of definitions, formation rules, and provability. It argues that:

- Definitions are acts of creating permitted axioms, not mere abbreviations.
- Traditional set-theoretic contradictions arise solely from inadequate formation rules.
- Gödel's incompleteness theorems lose relevance within this dynamic framework, since provability is emergent and depends on creative definitions.

A paradigm shift is proposed: mathematics as a living constructive process, where the frontier of provability evolves with the system.

1. Introduction

Classical mathematics has historically been conceived as a fixed formal system: a set of axioms and inference rules defining a static universe of provability and truth [1]. This framework has allowed significant formal advances but has also generated historical perceptions that now require reassessment:

- The incompleteness theorems of Kurt Gödel demonstrate inevitable limits for sufficiently expressive systems [2].
- Definitions have been considered conservative and merely abbreviations, unable to expand the formal space of provability [3].
- Classical set theory, subject to paradoxes (Russell, Burali-Forti, Cantor), has been interpreted as an ontological limit on the construction of mathematical objects [4].

ULOGIC proposes a paradigm shift: a **dynamic and constructive** formal system, where definitions are not mere abbreviations but active mechanisms of "axiom creation" (operationally, an axiom and a definition are the same: a new line in the argument that did not previously exist, introduced through a voluntary, creative, and unpredictable act). This approach allows rethinking provability, set-theoretic paradoxes, and the relevance of incompleteness theorems.

2. Definitions as Acts of Axiom Creation

2.1. The Classical View and Its Limitation

In traditional logic, a definition is considered *conservative*:

- It functions as an abbreviation that does not formally increase the set of provable theorems.
- Formally, for any theorem T in the original language, if it can be proven using definitions, there exists an equivalent proof without them.

However, this view ignores structural facts observable in mathematical practice:

- Theorems such as Cantor's ($|\mathcal{P}(A)| > |A|$) require essential intermediate definitions (e.g., the diagonal construction) [6].
- The absence of intermediate definitions makes many proofs unapproachable—not merely due to excessive complexity, but absolutely impossible to prove without these intermediate definitions.
- Each definition effectively introduces a new mathematical object that did not previously exist, as well as a new "line" that did not previously exist, functioning operationally as a permitted axiom.

2.2. The Historical Error of Treating Definitions as Abbreviations

Since Aristotle, the logical tradition has held that a definition is an abbreviation of already existing concepts: for example, the definition of a continuous function can be expressed as

$$\text{CONTINUOUS}(F) \iff F \text{ satisfies properties } P_1, P_2, \ldots, P_n.$$

From this perspective, "CONTINUOUS(F)" could formally be replaced by its expansion "F satisfies $P_1, \ldots, P_n$" in any context without altering the validity of proofs. This characterization of definitions as *eliminable abbreviations* has been considered the definition of a "good definition" for over 2,400 years [8].

However, starting with Cantor (1880) and the development of set theory, this view becomes untenable. Once the set of continuous functions is defined, expressions such as

$$\text{CLOSED}(\text{CONTINUOUS})$$

introduce new mathematical objects—the set CONTINUOUS—which cannot be eliminated by expanding the original definition. The word "CONTINUOUS" in this context is no longer an abbreviation: it represents an autonomous formal object with properties that can be studied, combining concepts such as "CLOSED" or "BOUNDED".

Consequently:

- Definitions cease to be eliminable and effectively operate as new axioms.
- The apparent "innocence" of classical definitions is a historical illusion that has allowed contradictions and confusion.

- The absence of explicit rules for creating definitions in mathematical practice is the root of many set-theoretic paradoxes.

ULOGIC formalizes this paradigm shift: definitions are consciously introduced new lines, equivalent to formal construction acts that generate previously nonexistent mathematical objects. This provides a consistent basis for constructing sets and proving properties without encountering unforeseen contradictions.

2.3. ULOGIC and the Formalization of Mathematical Creativity

ULOGIC encodes definitions as:

- **Operative acts of creating "new lines"**, which produce something previously nonexistent, rather than static abbreviations.
- **Licenses to construct new mathematical objects**, subject to controlled rules ensuring internal consistency.
- A system where provability depends on the definitions introduced and their interactions, making mathematics a dynamic and emergent process.

Paradigm shift: whereas classical logic views definitions as neutral expressive tools, ULOGIC elevates them to the core of mathematical content generation.

3. Set-Theoretic Contradictions and Formation Rules

3.1. Historical Errors in Set Theory

Set-theoretic paradoxes (Russell, Burali-Forti, Cantor) have been interpreted as ontological limitations of the concept of set [5]. Historically, this led to hierarchical systems, type theory, or artificial restrictions.

3.2. ULOGIC and the Naturalization of Sets

ULOGIC demonstrates that:

- Paradoxes arise solely from deficient definition rules.
- With precise rules for forming definitions, sets become consistent and natural.
- The notion of a set is derived from *consistent acts of definition*, not as a problematic ontological primitive.

Paradigm shift: set-theoretic problems are not inevitable, but rather a historical formal design error. ULOGIC enables a contradiction-free set theory based on construction and well-defined formation rules.

4. Gödel and Irrelevance in Dynamic Constructive Systems

4.1. Classical Interpretation

Gödel's incompleteness theorems state that:

- In any sufficiently expressive formal system, there exist true but unprovable theorems.
- This relies on an external notion of truth, typically interpreted via Tarskian models [7].

4.2. Critical Reassessment from ULOGIC

In ULOGIC:

- There is no external semantics: formulas are internal operative objects.
- Provability is dynamic and depends on which definitions have been introduced.
- Previously "unprovable" theorems may become provable through new definitions, continuously shifting the frontier of provability.

Consequence: Gödel's incompleteness loses practical and formal relevance in dynamic constructive systems like ULOGIC. Limits of provability are emergent, historical, and dependent on the creative flow of the system.

5. Mathematical Philosophy of ULOGIC

5.1. Mathematics as a Creative Process

ULOGIC proposes:

- Each definition is a creative act, a potential axiom expanding the mathematical space.
- Provability is emergent, not predictable a priori, depending on interactions between definitions and construction rules.
- Mathematics ceases to be a static universe of preexisting truths; it becomes an evolutionary space generated by construction.

5.2. Epistemological Consequences

- Mathematics is not measured only by fixed axiom systems, but should integrate formal creativity as a structural principle.
- The notion of truth is replaced by internal consistency and dynamic constructibility.

- An open, evolving mathematics is proposed, where the boundaries of provability depend on explicit creative decisions within the system.

6. Conclusion

ULOGIC redefines the foundations of real mathematics:

- Definitions are generative acts creating permitted axioms and expanding the space of provability.
- Set-theoretic paradoxes are not inevitable, arising only from inadequate definition rules.
- Gödel's incompleteness theorems are irrelevant to practical mathematics and to dynamic systems like ULOGIC.

This framework offers an innovative perspective: mathematics is not a static set of fixed truths, but a living, self-expanding constructive process, where the frontier of provability evolves with the introduction of definitions and creative axioms.

Referencias

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